

Notes:

Factor.

1. $2x^2 - 3x + 1$

$$(2x - 1)(x - 1)$$

2. $4x^2 - 9$

$$(2x - 3)(2x + 3)$$

3. $5x^2 + 6x + 1$

$$(5x + 1)(x + 1)$$

Rational Expression - the quotient of two polynomials.

Simplest Form - the numerator and denominator of a rational expression have no common factor

Examples:

1. What is $\frac{x^2 - 6x - 16}{x^2 + 5x + 6}$ in simplest form? State restrictions on the variable.

$$\frac{x^2 - 6x - 16}{x^2 + 5x + 6} = \frac{(x+2)(x-8)}{x^2 + 5x + 6} = \frac{(x+2)(x-8)}{(x+2)(x+3)} = \frac{x-8}{x+3}$$

$x \neq -3, -2$

2. What is the product $\frac{x^2 - 25}{x^2 + 4x + 3} \cdot \frac{x^2 + x - 6}{x - 5}$ in simplest form? State any restrictions on the variable.

$$\begin{aligned} \frac{x^2 - 25}{x^2 + 4x + 3} \cdot \frac{x^2 + x - 6}{x - 5} &= \frac{(x^2 - 25)(x^2 + x - 6)}{(x^2 + 4x + 3)(x - 5)} = \frac{(x+3)(x-2)(x^2 - 25)}{(x-5)(x^2 + 4x + 3)} \\ &= \frac{(x+3)(x-2)(x^2 - 25)}{(x-5)(x+1)(x+3)} = \frac{(x-2)(x^2 - 25)}{(x-5)(x+1)} = \frac{(x+5)(x-5)(x-2)}{(x-5)(x+1)} \\ &= \frac{(x+5)(x-2)}{x+1} \end{aligned}$$

$x \neq -3, -1, -5$

3. What is the quotient $\frac{x^2+5x+4}{x^2+x-12} \div \frac{x^2-1}{2x^2-6x}$ in simplest form? State any restrictions on the variable.

$$\begin{aligned} \left(\frac{(x^2+5x+4)}{(x^2+x-12)} \right) \div \left(\frac{(x^2-1)}{(2x^2-6x)} \right) &= \frac{(x^2+5x+4)(2x^2-6x)}{(x^2+x-12)(x^2-1)} = \frac{2x(x-3)(x+1)(x+4)}{(x^2-1)(x^2+x-12)} \\ &= \frac{2x(x-3)(x+1)(x+4)}{(x-3)(x+4)(x^2-1)} = \frac{2x(x+1)}{x^2-1} = \frac{2x(x+1)}{(x+1)(x-1)} = \frac{2x}{x-1} \end{aligned}$$

$x \neq -4, -1, 1, 3$

4. Find the product in simplest form of:

$$\frac{(2x^2+7x-15)}{(4x^2-8x+3)} \cdot \frac{(2x^2+x-1)}{(x^2+6x+5)}$$

$$\begin{aligned} \frac{(2x^2+7x-15)}{(4x^2-8x+3)} \cdot \frac{(2x^2+x-1)}{(x^2+6x+5)} &= \frac{(x+5)(2x-3)}{(2x-1)(2x-3)} \cdot \frac{(x+1)(2x-1)}{(x+1)(x+5)} \\ &= \frac{x+5}{2x-1} \cdot \frac{2x-1}{x+5} = \frac{(x+5)(2x-1)}{(2x-1)(x+5)} = \frac{2x-1}{2x-1} = 1 \end{aligned}$$

5. Find the quotient in simplest form of:

$$\frac{(12x^2-22x+8)}{(3x)} \div \frac{(3x^2+2x-8)}{(2x^2+4x)}$$

$$\begin{aligned} \frac{(12x^2-22x+8)}{(3x)} \div \frac{(3x^2+2x-8)}{(2x^2+4x)} &= \frac{(12x^2-22x+8)(2x^2+4x)}{3x(3x^2+2x-8)} \\ &= \frac{4x(x+2)(6x^2-11x+4)}{3x(3x^2+2x-8)} = \frac{4(6x^2-11x+4)(x+2)}{3(3x^2+2x-8)} = \frac{4(x+2)(2x-1)(3x-4)}{3(3x^2+2x-8)} \\ &= \frac{4(x+2)(2x-1)(3x-4)}{3(x+2)(3x-4)} = \frac{4(2x-1)}{3} \end{aligned}$$