What is a Radian?

Before we begin our investigation of a radian let us first establish a definition of an angle and review some important concepts from geometry.

Definition of an Angle:

A union of two rays with a common endpoint (vertex). **One side remains fixed** **and** **the other side rotates** **to form an angle.**

The side that remains fixed is called the initial side…

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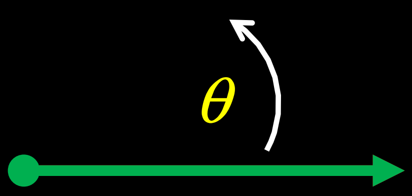

The side that remains fixed is called the initial side… and the side that rotates is called the terminal side.

We often use the Greek letter theta to represent an angle in trigonometry.

When the terminal side rotates in a counterclockwise direction the angle has a positive value.

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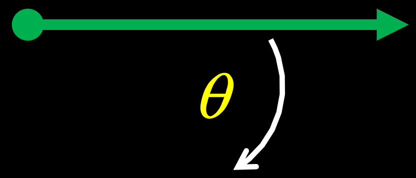

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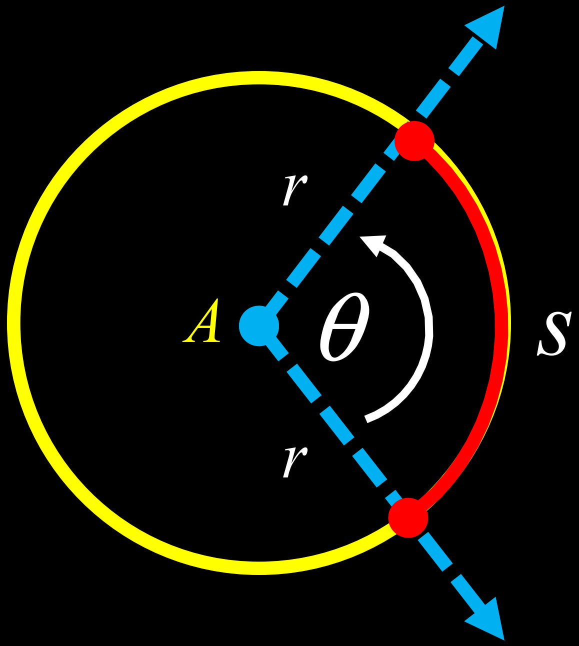
Geometry Review:

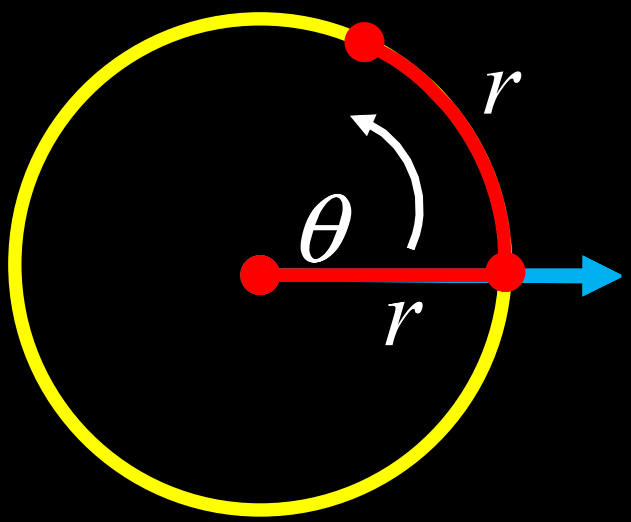


Given a circle with center *A*

- θ **is a central angle**
- r **is a radius**
- $2\pi r$ **is the circumference**
- s **is theta's intercepted arc**
- $s = \dfrac{\theta}{360^\circ} \cdot 2\pi r$ **(Arc Length)**

Now that we have our definition established and a refresher of some geometric concepts, we can discover the definition of a radian.

Radian: **In a circle, a central angle that cuts off an arc equal in length to the radius of the circle has a measure of 1 radian.**



When $s = r$ we have reached 1 radian

The measure of an angle (in radians) is found by dividing the arc length by the radius.

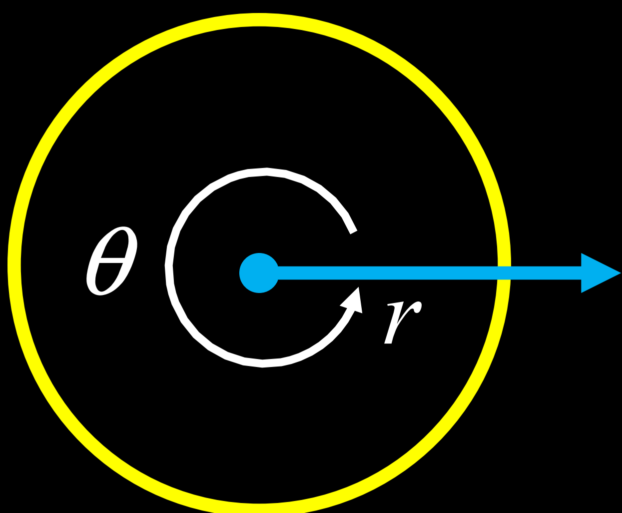
$$\theta_{(\text{in radians})} = \frac{\text{arc length}}{\text{radius of circle}}$$

$$\theta_{(\text{in radians})} = \frac{s}{r}$$

$$\theta = \frac{r}{r} \Rightarrow \theta = 1 \text{ radian}$$

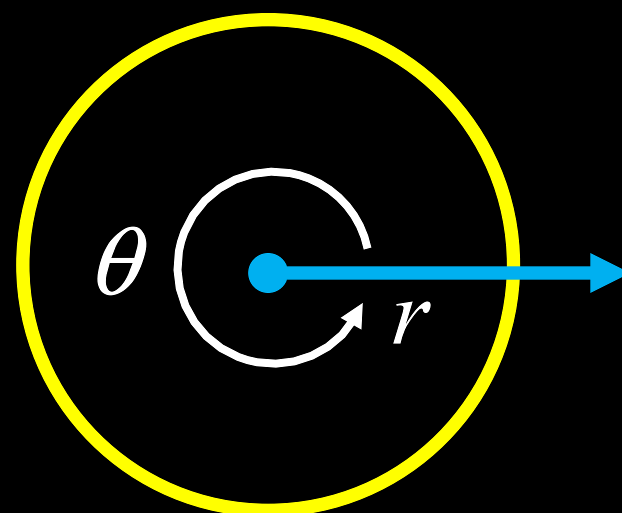
There exists a relationship between degree and radian measurements. Let us now explore that relationship.

One full rotation in degrees:



$$\theta = 360°$$

One full rotation in radians:



$$\theta = \frac{s}{r} \Rightarrow \theta = \frac{2\pi r}{r}$$

$$\theta = 2\pi \text{ radians}$$

The relationship between degrees and radians results in the following:

$$360 \text{ degrees} = 2\pi \text{ radians}$$ **(One rotation)**

$$180 \text{ degrees} = \pi \text{ radians}$$ **(Half rotation)**

Divide by 180 | **Divide by**

$$1 \text{ degree} = \frac{\pi}{180} \text{ radians}$$

$$\frac{180}{\pi} \text{ degrees} = 1 \text{ radian}$$

These tell me HOW to convert from one unit of measure to the other!

Summary:

- A radian is a unit of measure for angles. In a circle, a central angle that cuts off an arc equal in length to the radius of the circle has a measure of 1 radian.
- The measure of an angle (in radians) is found by dividing the arc length by the radius of a circle.
- To convert from degrees to radians, multiply the degree measure by...

$$\frac{\pi}{180}$$

- To convert from radians to degrees, multiply the radian measure by...

$$\frac{180}{\pi}$$

Guided Problems

- Guided problems applying the formulas

Convert to radians

1) $$210^\circ \cdot \frac{\pi}{180} = \frac{210\pi}{180} = \frac{7\pi}{6}$$

2) $$145^\circ \cdot \frac{\pi}{180} = \frac{145\pi}{180} = \frac{29\pi}{36}$$

Convert to Degrees

1) $\dfrac{5\pi}{3} \bullet \dfrac{180}{\pi} = \dfrac{900\pi}{3\pi} = 300°$

2) $\dfrac{7\pi}{12} \bullet \dfrac{180}{\pi} = \dfrac{1260\pi}{12\pi} = 105°$